ONE-DIMENSIONAL FLOWS IN ELECTROHYDRODYNAMICS

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Using the framework of electrohydrodynamics we study steady motions of a conducting medium composed of both, charged and uncharged components. Volume charge present in the medium and its interaction with the electric field, define the motion of the medium. Two cases are considered. In the first case we assume that the parameter Q ($Q = 4\pi q_0 L / E_0$) is small and the electric field given. General solution of the problem is obtained in the explicit form for the case when the field E_0 is constant, and a family of integral curves is constructed in the velocity-Mach number plane. If the velocity and Mach number are known at some point of the flow, we can use these curves to find the behavior of the flow downstream. In the second case we consider a flow in a variable electric field (arbitrary values of Q). A diagram constructed in the velocity-electric field plane makes possible a visual study of the motion in the case of a variable electric field. Singular points of the system of equations of the one-dimensional flow are investigated and the stability of the transonic passage is discussed.

One-dimensional equations in a constant electric field were investigated in [1] under a number of simplifying assumptions.

1. Fundamental equations. We consider a steady motion of a compressible, nonviscous medium without heat conduction, possessing volume charge in an electric field. We assume that all quantities depend on q only and that the electric field has a single component x. Motion in a channel of constant cross section in the case when the electric field component normal to the walls is neglected, is an example of such a flow.

For the problem under consideration, the equations of electrohydrodynamics will have the form

$$\rho u = m = \text{const}, \quad mu' + p' = qE, \quad m(c_p T + 0.5\, u^2)' = j_0 E \qquad (1.1)$$

$$E' = 4\pi q, \quad \varphi' = -E, \quad p = \rho RT, \quad j = q(u + bE) = j_0 = \text{const} \qquad (1.2)$$

where ρ denotes the density of the medium, u is the x-component of the velocity ($u > 0$), p is pressure, q is the electric volume charge density, E is the electric field strength, T is the temperature, j is the electric current density, c_p and R are the specific heat at constant pressure and the gas constant, respectively, and the prime denotes the derivatives with respect to x. We shall assume, for definiteness, that $q > 0$. For $q < 0$ the procedure is identical.

From (1.1) and (1.2) we obtain

$$u' = \frac{qEM^2(u - u_1)}{mu\,(M^2 - 1)}, \qquad M' = \frac{qEM^3(\gamma + 1)(u - u_2)}{2mu^2(M^2 - 1)}$$
$$u_1 = (\gamma - 1)\,bE, \quad u_2 = u_1\frac{1 + \gamma M^2}{\gamma + 1}, \quad \gamma = \frac{c_p}{c_v}, \quad M^2 = \frac{\rho u^2}{\gamma p} \qquad (1.3)$$

where M is the gasdynamic Mach number and b is mobility.

2. Flow in a constant electric field. First equation of (1.2) can be written as $E_*' = Qq_*$, $\quad Q = 4\pi q_0 L / E_0$, $\quad E_* = E / E_0$, $\quad q_* = q / q_0$

where L is the characteristic length. We shall consider the case of small Q, assuming $E = E_0 = \text{const}$. Then, taking the expression for the charge density q from the last equation of (1.2), we obtain a closed system of equations in u and M.

Introducing the dimensionless velocity $U = u / |u_1|$ and eliminating x from (1.3), we obtain

$$\frac{dM}{dU} = \frac{(\gamma+1) M (U - U_2 \operatorname{sign} E_0)}{2U(U - \operatorname{sign} E_0)}, \quad U_2 = \frac{(1+\gamma M^2)}{\gamma+1} \tag{2.1}$$

General solution of (2.1) has the form

$$M^2 = \frac{U |U - \operatorname{sign} E_0|^\gamma}{|U - \operatorname{sign} E_0|^\gamma + C}, \quad C = \frac{(U_0 - M_0^2) |U_0 - \operatorname{sign} E_0|^\gamma}{M_0^2} \tag{2.2}$$

where U_0 and M_0 are the values of the parameters at cross section $x = 0$.

General solution of (1.3) with (2.2) taken into account is given by

$$x = \frac{m u_1^2}{(\gamma-1) E_0 j_0} \left\{ \frac{1}{2} (U - U_0) [(\gamma-1)(U + U_0) + 2] + \right.$$

$$\left. + C \operatorname{sign}(U_0 - \operatorname{sign} E_0) \left[\frac{|U - \operatorname{sign} E_0| + 1}{|U - \operatorname{sign} E_0|^\gamma} - \frac{|U_0 - \operatorname{sign} E_0| + 1}{|U_0 - \operatorname{sign} E_0|^\gamma} \right] \right\} \tag{2.3}$$

Equations (2.2) and (2.3) define the dependence of the velocity and the Mach number on x.

The solution obtained can be conveniently investigated in the UM-plane.

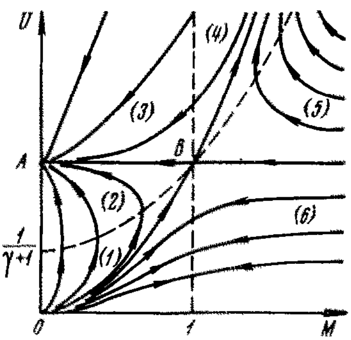

Fig. 1

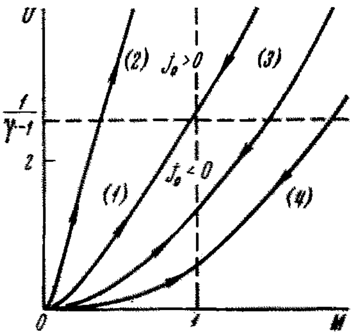

Fig. 2

Figures 1 and 2 depict the integral curves of Eq. (2.1) for $E_0 > 0$ and $E_0 < 0$, respectively. The field of integral curves depends on a single parameter γ. Passage along the integral curves in directions indicated by arrows, corresponds to the motion downstream.

Let us consider the case $E_0 > 0$ (Fig. 1). The straight line $U = 1$ and the lines $M = 1$ and $U = U_2$ shown by the broken lines, divide the plane into six regions. Points $O(0, 0)$, $A(0, 1)$ and $B(1,1)$ are the singular points of (2.1). In Sect. 4 we show that the points O and A are nodes, and B is a saddle point. The lines $U = 0$ and $M = 0$ represent particular solutions at the point O, the lines $U = 1$ and $M = 0$ – at the point A, and the lines $U = M^2$ and $U = 1$ – at the point B. We note that the line $U = M^2$ also passes through O, and the line $U = 1$ connects the singular points A and B.

If in some cross section of the channel the velocity and Mach number have such values that the corresponding points on the UM-plane are situated in region 6 and in the part of region 1 below the integral curve $U = M^2$, then the Mach number may reach the value of unity during the passage downstream. This may, however, happen only at the end of the channel. In region 4 the gas may be accelerated indefinitely. Integral curves lying in the regions 2 and 3 terminate at the point A.

All points of intersection of the line $U = U_2$ with the integral curves with exception of the point B are such, that the tangents to the integral curves are vertical at these points.

If the initial values lie on the line $U = M^2$ (when $M < 1$) or on $U = 1$ (when $M > 1$), a continuous passage through the sound velocity may take place in the motion along the channel. In the first case it is possible to pass from the subsonic to the subsonic or supersonic mode, and in the second case – from the supersonic to the subsonic or supersonic mode. The outcome depends on the boundary conditions at the channel exit. In Sect. 4 we show, that all modes in which a continuous passage takes place at the point B, are stable. Formula (2. 3) yields the distance from the channel entry to the point at which such a passage occurs. For the first and second case we have, respectively,

$$x = \frac{mu_1^2}{2j_0E_0(\gamma-1)}(1-U_0)[(\gamma-1)U_0+\gamma+1] \qquad (M_0<1)$$

$$x = \frac{mu_1^2}{j_0E_0(\gamma-1)}\left(1-\frac{1}{M_0^2}\right) \qquad (M_0>1)$$

Let us consider the case when $E_0 < 0$. The lines $U = 1/(\gamma - 1)$ and $M = 1$ divide the UM-plane into four regions. Electric current density is positive above the line $U = 1/(\gamma - 1)$ and negative below it. Regions 2 and 3 correspond to the generating mode ($j_0E_0 < 0$, $j_0 > 0$), and regions 1 and 4 – to the accelerating mode ($j_0E_0 > 0$, $j_0 < 0$). Since in the given formulation $j = j_0 = \text{const}$, the passage from the accelerating to the generating mode and vice versa, is impossible. Let us recall that $q > 0$. Figure 2 shows that under certain conditions a motion is possible in the regions 2 and 3 such, that $u \to -bE_0$. From the last equation of (1. 2) it follows, that in such a motion $q \to \infty$ (u', M' and E' also tend to infinity).

If we consider a one-dimensional flow in a channel of arbitrary length, then the channel exit should correspond to the point at which the velocity assumes the value $u = -bE_*$. If the channel length is given, the correctness of the formulation of the problem should be ensured by the proper choice of the boundary conditions. The problem of formulation of the boundary conditions for the system (1. 3) is discussed in Sect. 3.

The case $j_0 = 0$ corresponds to a gasdynamic flow with constant parameters.

We note that the case $j_0 = 0$ also describes a flow in an EHD generator in the standby condition, and we have $q = 0$ and $u_0 = -bE_0 = \text{const}$. The potential difference over the length L of the generator is $\Delta\varphi = -E_0L = u_0L/b$.

3. Flow in a variable electric field. We shall consider the case of arbitrary Q. We can integrate the last two equations of (1. 1), introducing the electric field potential φ and eliminating the charge q with help of the first equation of (1. 2). Then we will have

$$\rho u = m = \text{const}, \quad mu + p - E^2/8\pi = \Pi = \text{const} \tag{3.1}$$

$$m(c_pT + 0.5\ u^2) + j_0\varphi = \varepsilon = \text{const} \tag{3.2}$$

Using the third equation of (1. 2) we obtain from the above equations

$$u^2 - \frac{2\gamma(\Pi + E^2/8\pi)}{(\gamma+1)m}u + y = 0, \quad y = \frac{2(\gamma-1)}{m(\gamma+1)}(\varepsilon - j_0\varphi) \tag{3.3}$$

which describes a third order surface in the uEy-space, containing the integral curves of the initial system. However, only that part of the surface on which $u \geqslant 0$ and $y \geqslant 0$, is physically meaningful. Translation along the surface (3.3) corresponds to a motion along the channel. Although this surface can easily be constructed in the uEy-space, various modes of flow can be studied conveniently in the case of a variable electric field in the uE-plane, projecting a part of the surface (3.3) on the plane $y = 0$ (Fig. 3).

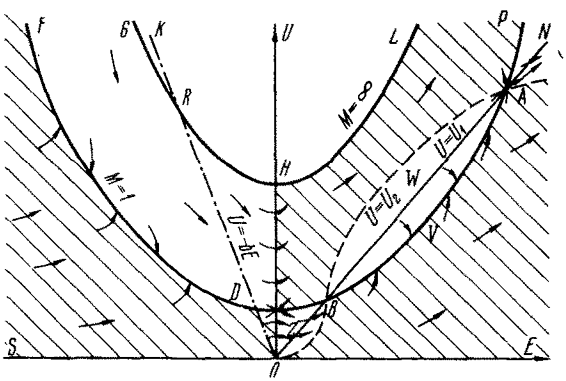

Fig. 3

Equations describing the behavior of the integral curves on the uE-plane have the form

$$\frac{du}{dx} = \frac{j_0 E\,[u - (\gamma - 1)\,bE]}{(u + bE)\,[(\gamma + 1)\,mu - \gamma\Pi - \gamma E^2/8\pi]}, \quad \frac{dE}{dx} = \frac{4\pi j_0}{u + bE} \tag{3.4}$$

First equation of (1.3) and the relation

$$M^2 = \frac{mu}{\gamma\,(\Pi + E^2/8\pi - mu)} \tag{3.5}$$

were used in constructing the first equation of (3.4).

Formula (3.5) follows from the second equation of (3.1). Eliminating the variable x from (3.4) we obtain

$$\frac{du}{dE} = \frac{E\,[u - (\gamma - 1)\,bE]}{4\pi\,[(\gamma + 1)\,mu - \gamma\Pi - \gamma E^2/8\pi]} \tag{3.6}$$

Below we shall only consider the case when $\Pi > 0$, since in the case of $\Pi < 0$ the procedure is analogous.

Figure 3 depicts the parabolas $M = 1$ (line $ABCDF$) and $M = \infty$ (line LHG). Equations of these parabolas follow from (3.5) and are, respectively,

$$u = \frac{\gamma}{m(\gamma + 1)}\,(E^2/8\pi + \Pi), \quad u = (E^2/8\pi + \Pi)/m \tag{3.7}$$

The region contained between the straight line $u = 0$ and the upper parabola LHG corresponds to a real flow in the uE-plane. The flow is supersonic ($M > 1$) in the region above the parabola $PABCDF$ and subsonic ($M < 1$) below it. In addition Fig. 3 shows the lines $u = -bE$ (line OK) and $u = u_1 = (\gamma - 1)\,bE$ (line ON). Relative positions of the parabolas $M = 1$ and $M = \infty$ and the lines $u = u_1$ and $u = -bE$ may vary according to the parameters γ, Π, m and b. Coordinates of the points of intersection of the line $u = u_1$ and the parabola $M = 1$ are

$$E_A = d + \triangle^{1/2}, \; E_B = d - \triangle^{1/2}, \; \triangle = d^2 - 8\pi\Pi, \; d = 4\pi(\gamma^2 - 1)\;mb/\gamma$$

$$u_{A,B} = (\gamma - 1)\;bE_{A,B}, \; E_{A,B}, \geqslant 0 \tag{3.8}$$

Figure 3 illustrates the case when $\triangle > 0$, where we have two points of intersection. When $\triangle = 0$, then the line $u = u_1$ touches the parabola $M = 1$, and when $\triangle < 0$, there are no common points. We note that the line $u = u_1$ can also intersect the parabola $M = \infty$. On crossing the line $u = u_1$ and the parabola $PABCDF$ we find, that the derivative du/dE changes its sign everywhere except at the points A and B where it needs not. On the line $u = u_1$ the derivative du/dE becomes zero, and on the parabola $PABCDF$ it becomes infinite. Points of intersection A and B are singular points of Eq. (3.6) as well as the point C at which both, the numerator ($E = 0$) and the denominator ($M = 1$) of (3.6) vanish. The current density is $j_0 < 0$ to the left of the line OK, and $j_0 > 0$ to the right. Passage across the line OK is impossible for reasons given in Sect. 2.

The line $u = u_2 = u_1(1 + \gamma M^2) / (\gamma + 1)$ is shown in Fig. 3 by dashes, and $dM / dE = 0$ everywhere on this line except at the points A and B. In the shaded areas, $dM / dE > 0$. The lines appearing on the figure divide the uE-plane into seven regions in which various modes of flow can be realized. When $E > 0$, we have $M < 1$ in the regions $OCBZ$, $OZBVANE$, PAN, $M > 1$ in $CBWAPLH$ and $BVAW$; when $E < 0$, we have $M < 1$ in $FDCOS$ and $M > 1$ in $GHCDF$.

The second equation of (3.4) implies that the electric field increases as we move along the channel. With the velocity and the electric field given at some cross section of the channel, the behavior of u and E during the motion that follows, can be described by (3.6) and is indicated by arrows in Fig. 3. The value of the Mach number equal to unity may be reached on moving along the channel (motion in the uE-plane in directions indicated by arrows), the arrows touch the parabola $M = 1$ in the uE-plane. If at the same time the integral curves do not pass through the singular points A, B or C, then the above situation can only occur at the channel exit. Continuous passage through the value $M = 1$ at the points distinct from A, B or C, is impossible.

It is shown in Sect. 4 that the singular points A and C are saddle points. In general, the passage from subsonic mode to supersonic and vice versa are possible at these points. The singular point B can either be a focus, or a node, depending on the values of γ, Π, m and b. In the first case the integral lines approaching the point B must intersect the curve $M = 1$ at the point different from B, and this corresponds to the channel exit. In the second case, the integral curves may enter the point B and a continuous passage through the sonic speed may also take place.

The parabola $M = \infty$ is unattainable at finite value of gas velocity. The gas may be accelerated indefinitely in the regions $CBWAPLH$ and $OZBVANE$, and in the latter region the acceleration occurs at $M < 1$. Region $ODRHC$ corresponds to the generating mode ($j_0 E < 0$) in the uE-plane, in the remaining regions the accelerating mode takes place.

We note that flows with discontinuities can be constructed in certain cases. Under the present formulation these will, generally, be gasdynamic discontinuities at which the electric field is constant and the condition $j_0 = \text{const}$ holds. The corresponding passage in the uEy-space will be the passage from the part of the surface (3.3) at which $M > 1$, to that part at which $M < 1$

If the constants j_0, γ, b, m and Π together with the boundary conditions

$$u = u_0, \quad E = E_0 \quad \text{when } x = 0 \tag{3.9}$$

at the channel entry are given, then the Cauchy's problem (3.4) and (3.9) can be solved to yield the velocity and electric field distribution along the channel. We may however find, that at a certain cross section of the channel either the $M = 1$ or the velocity $u = -bE$ is reached. If the formulation of the problem leaves the channel length undefined, it can be assumed that the channel exit corresponds to this cross section. If, on the other hand, the channel length is given, we may find that the cross section in question is reached at $x < L$ (where L denotes the channel length), i.e. that the Cauchy's problem is formulated incorrectly. Formulations of the problem leaving E_0 and Π unknown, are possible. Such a situation arises e.g. when the boundary conditions for the electric field potential are given

$$\varphi = 0 \quad \text{when} \quad x = 0, \; \varphi = \varphi_1 \; (\text{or} \;\; \partial\varphi / \partial x = 0) \quad \text{when} \;\; x = L$$

When the Cauchy's problem (3.4) and (3.9) has no solution, a boundary value problem must be constructed for the system (3.4) similarly to what is done in gas dynamics. We can e.g. define the values of the velocity and the Mach number at the channel exit

$$U = U_L = -\,kbE_L, \quad M = M_L \quad \text{when} \;\; x = L$$

Here $k < 1$ for $j_0 < 0$ and $k > 1$ for $j_0 > 0$. In addition, either the relevant number of conditions must be set up at the channel entry, or some of the constants Π, j_0, m, etc. must be assumed definable by the solution of the problem.

In the case of a constant electric field, the boundary conditions are formulated in a similar manner.

4. Investigation of singular points of the one-dimensional flow equations. When the electric field is constant, i.e. $E = E_0$, the singular points can conveniently be investigated in the UM-plane. Equation (2.1) has three singular points there, at which the numerator and the denominator of the right side of (2.1) vanish simultaneously. Coordinates of these points are given in Sect. 2 (points O, A and B).

Points O and A are nodes [2]. Indeed, the characteristic equation at O is

$$\lambda^2 - \alpha\lambda - \beta = 0, \quad \alpha = 3, \quad \beta = -2 \tag{4.1}$$

Roots of (4.1) are real and of equal sign ($\lambda_1 = 1$, $\lambda_2 = 2$). At the point A the roots of the characteristic polynomial are $\lambda_1 = 1$ and $\lambda_2 = 2 / \gamma$.

Point B is a saddle point. Indeed, the characteristic equation at B has the form

$$\lambda^2 - \alpha\lambda - \beta = 0, \quad \alpha = -2\,(\gamma - 1), \quad \beta = 4\gamma \tag{4.2}$$

Its roots are real and of opposite sign: $\lambda_1 = 2$ and $\lambda_2 = -2\gamma$. Two integral curves passing through B exist and are given in Sect. 2.

Study of the stability of the passage through the sound velocity at B requires the characteristic equation in the Mx-plane. It can easily be shown that it has the form

$$\lambda^2 - \alpha\lambda + \beta = 0, \quad \alpha = -S\,(\gamma - 1), \quad \beta = -\gamma S^2, \quad S = \frac{(\gamma - 1)\, j_0 E_0}{\gamma m u_1^2} > 0$$

From it we see that the singular point in the Mx-plane is also a saddle point ($\beta < 0$). Utilizing the results of [3] we can conclude that the continuous passage through the sound velocity along the particular solutions is stable, since α in the characteristic equation is negative.

When the electric field is variable, Eq. (3.6) may have one, two or three singular points in the uE-plane; it depends on the values of parameters γ, Π, m and b. Figure 3 depicts the case of three such points, namely A, B and C. Coordinates of C are $E_c = 0$, $u_c = \gamma\Pi / m\,(\gamma + 1)$, the coordinates of A and B are given in Sect. 3 (see (3.8)).

Point C is a saddle point, since the characteristic equation at C has the form

$$\lambda^2 - \beta^2 = 0, \quad \lambda_{1,2} = \pm\,\beta, \quad \beta = [4\pi\,(\gamma + 1)\; m u_c]^{1/2}$$

its roots being real and of opposite sign.

Two integral curves passing through the point C exist (Fig. 3), along which a continuous passage of the flow from the subsonic to the supersonic or subsonic mode, and from the supersonic to the subsonic or supersonic mode is possible. Singular points A and B

are distinct, when the discriminant $\Delta > 0$ (see (3. 8)). Charcteristic equation at the point A is

$$\lambda^2 - \alpha\lambda + \beta = 0, \ \alpha = -(\gamma - 1) E_A, \ \beta = -\gamma E_A \Delta^{1/2} \tag{4.3}$$

its roots are real, and are of opposite sign.

It follows, that the singular point A is a saddle point. Two integral curves pass through it and a continuous passage from the supersonic flow region to the subsonic or supersonic, and from the subsonic region to the supersonic or subsonic region, is possible on these curves.

Point B can either be a node or a focus, depending on the values of parameters γ, Π, m and b. The characteristic equation at B has the form

$$\lambda^2 - \alpha\lambda - \beta = 0, \ \alpha = -(\gamma - 1) E_B, \ \beta = -\gamma E_B \Delta^{1/2} \tag{4.4}$$

It can easily be shown that B is a focus when

$$\frac{c}{d^2} < \frac{8\gamma(\gamma^2+1)}{(\gamma+1)^4}, \quad d = \frac{4\pi(\gamma^2-1)\,mb}{\gamma}, \quad c = 8\pi\Pi$$

Figures 4 and 5 show two possible arrangements of the integral curves when B is a focus. Continuous passage through the sound velocity at the point B is impossible, since the line $M=1$ is reached during the approach to B.

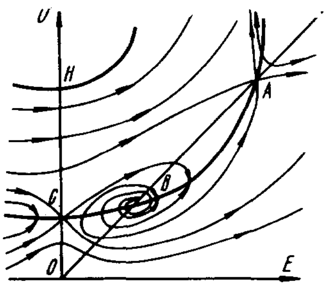

Fig. 4

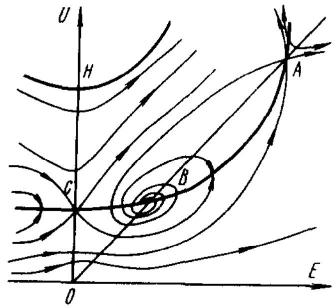

Fig. 5

If $1 > c / d^2 > 8\gamma(\gamma^2 + 1) / (\gamma + 1)^4$, then B will be a node at which the continuous passage is possible. Analyzing the exclusive directions we find, that the only possible continuous passage is that from the supersonic to subsonic region.

To investigate the stability of passages through the sound velocity at A and B, we must write the characteristic equation at these points in the Mx-plane. This is easily done using (3. 5) and expanding the electric field magnitudes near the singular point into a series: $E = E_{A,B} + 4\pi q_{A,B} \cdot (x - x_{A,B})$ where $E_{A,B}, q_{A,B}, x_{A,B}$ denote the values of the parameters at the singular point.

In the Mx-plane we have

$$\lambda^2 - \alpha\lambda - \beta = 0, \quad \alpha = -\frac{q_{A,B}}{mb}, \quad \beta = \frac{\gamma\alpha^2}{(\gamma-1)^2 E_{A,B}}\left[E_{A,B} - 4\pi\frac{(\gamma^2-1)\,mb}{\gamma}\right]$$

from which it follows that in the Mx-plane the point A will be a saddle point, and can either be a node, or a focus. Since B in the characteristic equation is negative, the continuous passage through the sound velocity is stable at A and B [3].

Singular points A and B are absent when $\Delta < 0$. When $\Delta = 0$, the points A and B merge into a single point which, as seen from (3. 6), is a degenerate saddle point.

One-dimensional motion in electrohydrodynamics at large electric Reynolds numbers was dealt with in [4].

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DIFFRACTION OF A CYLINDRICAL HYDROACOUSTIC WAVE AT THE JOINT OF TWO SEMI-INFINITE PLATES

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The two-dimensional stationary diffraction problem is considered. A fluid medium fills the lower half-plane in which acoustic effects are generated by a point type source located at a certain depth. The surface of the fluid is covered by two abutting semi-infinite plates. Mechanical properties of the two plates are assumed to be different. An exact mathematical solution of the problem is constructed for the case in which conditions at the plates abuttment are not fixed. This solution (which we shall call "general") contains a certain number of arbitrary constants. The method for determination of these constants for specified conditions at the joint is indicated. A characteristic of the latter problem is that formal application of the boundary contact operators to the general solution generates divergent integrals of expressions which increase algebraically at infinity.

The analysis is carried out in certain abstract terms . The expressions of boundary, and boundary contact operators are not specified, hence these results are valid for the various methods used in plate theory approximations. The derived solutions may also be used for other boundary conditions (e.g. when one part of the fluid surface is left free, or covered by a membrane).

1. Formulation of problem. A compressible fluid fills the lower half-plane $(-\infty < x < +\infty,\ 0 < y < +\infty)$. Two semi-infinite plates lie on the surface of the fluid ($y = 0$) extending respectively in the positive and negative directions of the x-axis (Fig. 1). The field generated in the described system by a point source of harmonic oscillations (at point x_0, y_0) is to be defined. Factor $e^{-i\omega t}$ defining the dependence of processes on time will be everywhere omitted.

We shall describe the acoustic processes in the fluid in terms of pressure $P(x, y)$. The problem as stated consists of finding a solution of the inhomogeneous Helmholtz' equation

$$\Delta P + k^2 P = \delta(x - x_0,\ y - y_0) \qquad (-\infty < x < +\infty,\ \ 0 < y < +\infty) \qquad (1.1)$$

with boundary conditions

$$L_1 P = 0 \quad (x > 0), \qquad L_2 P = 0 \quad (x < 0) \qquad (1.2)$$